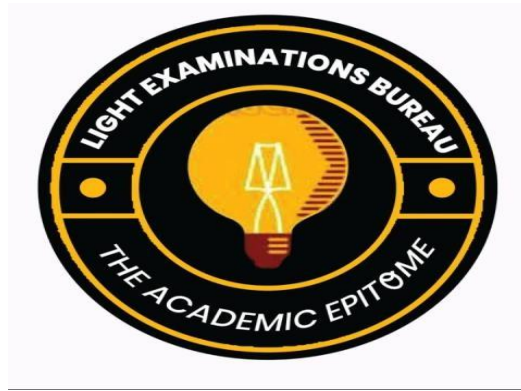




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***VISION: *** To be the Supreme Quality Examining body in Uganda.

***MISSION: *** To produce Competent Citizens of Uganda that have through Quality Assessment and can create a Positive Impact in their Communities.

***MOTTO: *** The Academic Epitome.

***SLOGAN: *** We set, they respond.

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***LIGHT EXAMINATIONS BUREAU(LEB)’S A-LEVEL
MATHEMATICS SEMINAR HELD AT ST. THOMAS VOCATIONAL
SECONDARY SCHOOL-RUBIRIZI ON SATURDAY- 12TH-JULY-2025 *
TRIGONOMETRY**

1 (a) Without using a calculator show that

(i) $\cos 36^\circ - \cos 72^\circ = \frac{1}{2}$

(ii) $\tan^{-1} \frac{1}{4} + \tan^{-1} \frac{3}{5} = \frac{\pi}{4}$

(b) Solve the equations for θ in the range $0^\circ \leq \theta < 360^\circ$

(i) $2 \cos^2 \theta + \sin \theta = 1$

(ii) $\sin \theta + \sin 2\theta + \sin 3\theta = 0$

(c) Express $4 \cos \theta - 3 \sin \theta$ in the form $R \cos(\theta + \alpha)$ where R is a positive constant and α is an acute angle. Hence find the minimum value of the expressions. Deduce also the smallest value of θ for which the minimum occurs.

(i) $f(x) = 9 + 4 \cos \theta - 3 \sin \theta$

(ii) $g(x) = \frac{1}{7 + 4 \cos \theta - 3 \sin \theta}$

2 (a) Solve the equations for θ the interval $0^\circ \leq \theta \leq 360^\circ$

(i) $\sin \theta + 2 \sin(\theta + 120^\circ) + \sin(\theta + 240^\circ) = 0$

(ii) $\cos 2\theta + \cos 4\theta = 1$

(b) If $\sin A = \frac{4}{5}$ and $\cos B = \frac{3}{5}$ when A is obtuse and B is acute. Find the value of

(i) $\sin(A - B)$

(ii) $\tan(A + B)$

(c) In any triangle ABC prove that

(i) $\sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$

$$(ii) \cos A + \cos B + \cos C = 1 + 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$$

3 (a) Solve the equations for θ in the interval of $-180 < \theta \leq 180^\circ$

$$(i) 2 \cos(\theta + 30^\circ) = 3 \sin(\theta - 30^\circ)$$

$$(ii) 2 \tan \theta + 3 \sec \theta = \cos \theta$$

(b) Determine the Cartesian equation represented by the equations

$$(i) x = \cos \theta, y = 3 \sin 2\theta$$

$$(ii) x = 4 \tan \theta + 3 \sec \theta = \cos \theta, y = \tan \theta - \cos \theta$$

$$(c) \text{ Prove that (i) } \frac{\cos \theta - 1}{\sec \theta + \tan \theta} + \frac{\cos \theta + 1}{\sec \theta - \tan \theta} = 2(1 + \tan \theta)$$

$$(ii) \tan^2 \theta - \sin^2 \theta = \sin^4 \theta \sec^2 \theta$$

4 (a) Solve the triangle in which $c = 26.83$, $A = 80.5^\circ$ and $B = 42.8^\circ$

(b) Solve the equations θ in the interval of $0 < \theta \leq 2\pi$

$$(i) 5 \cos \theta + 3 \sin \theta = 4$$

$$(ii) 7 \cos^2 \theta - 2 \sin \theta \cos \theta - 3 \sin^2 \theta = 1$$

$$(c) \text{ Prove that } \sin \theta \sin(60 - \theta) \sin(60 + \theta) = \frac{1}{4} \sin 3\theta$$

$$(ii) \text{ Prove that } \sin \theta = \frac{2t}{1+t^2}, \text{ where } t = \tan \frac{\theta}{2}. \text{ Hence show that } \tan 15^\circ = 2 - \sqrt{3}.$$

(d) Show that $\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$, find the values of x satisfying the equation $8x^3 - 6x + 1 = 0$. Correct to 3 decimal places.

5 (a) Without using a calculator show that $\frac{\cos 29^\circ + \sin 29^\circ}{\cos 29^\circ - \sin 29^\circ} = \tan 74^\circ$

(b) Given that $p = \cos A - \cos B$ and $q = \sin A - \sin B$. Express in terms of p and q

$$(i) \sin(A + B)$$

$$(ii) \cos(A - B)$$

(c) Solve the equations for θ in the range $0^\circ < \theta \leq 360$

$$(i) \cos \theta \cos 3\theta - \sin \theta \sin 3\theta = 1$$

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(ii) $2 \cos^2 3\theta = 3(1 - \sin 3\theta)$

(d) Prove that in any triangle ABC : $\left(\frac{b-c}{b+c}\right) \tan \frac{1}{2}(B+C) = \tan \frac{1}{2}(B-C)$

ANALYSIS

6 (a) Differentiate from first principles.

(i) x^2

(ii) $\frac{2}{\sqrt{x}}$

(iii) $\sin 2x$

(iv) e^{2x}

(b) Differentiate with respect to x

(i) $x^2 + 6x + 5$

(ii) e^{2x+3}

(iii) $\sqrt{\cos 2x}$

(c) Use differential calculus to approximate the value of

(i) $\sqrt{9.8}$

(ii) $\sqrt[3]{8.09}$

(iii) $\sin 59.5^\circ$

(d) On the same axes sketch the curves $f(x) = x^2 - 4x + 3$ and $y = \frac{1}{f(x)}$

State and illustrate the asymptotes on the curve.

7 (a) Evaluate the integrals.

(i) $\int x^3 + 3x + 4 dx$

(ii) $\int \frac{\sin x dx}{1 - \cos x}$

(iii) $\int \frac{\cos x dx}{1 + \sin^2 x}$

(b) Sketch on the same axes the curve $y = x^2 - 3x + 2$ and the line $3x + y = 6$

(i) Find the area bounded by the curve

(ii) Determine the volume of the solid formed when the area enclosed by the curve and the line are rotated about the x -axis through 360° .

(c) Use Maclaurin's theorem to expand the functions in ascending powers of x upto x^2

(i) $\frac{1}{\sqrt{1-x}}$

(ii) $\ln\left(\frac{1+2x}{1+x}\right)$

(iii) e^{x^2+2x}

8 Find the derivatives of the following functions.

(i) $\frac{e^{2x} \sin x}{(1+x^2)(1-x)}$

(ii) $x^3 \log_{10}(1+x^2)$

(iii) $x^2 + x^{\sin x}$

(b) Determine the equation of the tangent and normal to the curve at the given points.

$$(i) y = x^2 - 3\log_2 x + 5 \quad (2,6) \qquad (ii) y = \frac{x-2}{x+1} \quad (-2,4)$$

(c) A 6% error is made in measuring the radius of the sphere, find the percentage error made in the volume of the sphere.

(d) A rectangular farm is to be fenced using a wire mesh of 2000m with one side an existing straight wall. Find the maximum area that can be enclosed and the corresponding dimensions.

9 (a) Evaluate the following integrals

$$(i) \int_0^{\frac{\pi}{2}} x \sin x dx$$

$$(ii) \int_0^2 \frac{dx}{4+x^2}$$

$$(iii) \int_0^{\ln 2} \frac{e^x dx}{1+e^x}$$

(b) Solve the differential equations

$$(i) \frac{dy}{dx} + y = e^{2x}$$

$$(ii) \frac{dy}{dx} = \frac{x \ln x}{y}$$

$$(iii) x \frac{dy}{dx} + y = \cos x$$

(c) The gradient function of the curve is given by $x + y$. The curve passes through the point $(0,3)$. Find the

- (i) Equation of the curve
- (ii) Value of y when $x = \ln 2$.

10 (a) Differentiate with respect to x

$$(i) \log_e \left[\frac{(1+x^2)(4-x)^2}{\sqrt{1-x^2}} \right]$$

$$(ii) \tan^{-1} \left(\frac{1+x}{1-x} \right)$$

$$(iii) \frac{x \sin x}{(1+e^{2x})}$$

(b) Differentiate 4^{2x+1} with respect to x , hence $\int 4^{2x+1} dx$

(c) A curve is given parametrically by the equations.

$$x = t + \frac{1}{t}, \quad y = t - \frac{1}{t}.$$

(i) Find the Cartesian equation of the curve.

(ii) Determine $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of t .

(iii) Find the equation of the tangent at the point $\left(\frac{5}{2}, \frac{3}{2}\right)$.

11 (a) If $y = \frac{\cos x}{x}$, prove that $\frac{xd^2y}{dx^2} + \frac{dy}{dx} + xy = 0$

(b) A curve is given parametrically by the equations

$$x = t^3 - 3t + 4, y = t^2 - 4t - 1.$$

(i) Find $\frac{dy}{dx}$ in terms of t

(ii) Determine the coordinates of the turning point of the curve.

(c) A curve is given by the parametric equations

$$x = \frac{1+t}{2+t}, y = \frac{t}{3+t}.$$

(i) Find the Cartesian equation of the curve.

(ii) Sketch the curve.

12 (a) Given that $y = \sqrt{\frac{1+\sin x}{1-\sin x}}$, show that $\frac{dy}{dx} = \frac{1}{1-\sin x}$

(b) Find (i) $\int_0^{\frac{\pi}{2}} x \cos(x^2) dx$ (ii) $\int_0^{\frac{\pi}{4}} \cos^3 x \sin x dx$ (iii) $\int_0^{\frac{\pi}{2}} \frac{5d\theta}{3+4\cos\theta}$

(c) Find the turning points of the curve $y = \frac{x^2 - 4}{(x-1)^2}$.

(ii) State the asymptotes of the curve

(iii) Sketch the curve $y = \frac{x^2 - 4}{(x-1)^2}$

13 (a) Differentiate with respect to x

(i) $y = x^3 + e^{3x} + 3^x$ (ii) $y = x^2 + \sin x$ (iii) $y = x^3 + (\ln x)^x$

(b) If $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ show that $\frac{dy}{dx} = \frac{2}{1+x^2}$.

(c) An inverted circular cone of base radius 12cm and height 36 cm is being filled at a constant rate of $8\pi \text{ cm}^3/\text{min}$. Find the rate of increase of the water level when the height is 24cm.

(d) For the curve $y = \frac{x^2}{x-3}$.

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- (i) Find the range of values of y for which the curve cannot exist for real values of x
- (ii) Determine the nature of the turning points of the curve.
- (iii) State the asymptotes of the curve. Hence sketch the curve.

14 (a) Differentiate with respect to x

(i) $\frac{xe^{2x} \sin x}{\sqrt{(1+x^2)^3}}$

(ii) $\log_{10} \left[\frac{(1-x)^2 \sqrt{(1+x)^3}}{(1+x^2)^3} \right]$

(b) Find the integrals

(i) $\int_0^{\frac{\pi}{2}} \sin 5x \cos 3x dx$

(ii) $\int_0^1 x^5 e^{x^3} dx$

(iii) $\int_1^{\sqrt{3}} \frac{x^2 dx}{\sqrt{x^2 - x^4}}$

(c) A piece of wire 0.1m long is cut into two parts one of which is bent into a circular and the other into a square. If the sum of the areas of the circle and of the square is to be maximum, find the radius of the circle.

15 (a) Resolve $f(x) = \frac{2x^3}{(x-1)(x^2-1)}$ into partial fractions

(ii) Find $\int f(x) dx$

(b) The rate at which the temperature of the body falls is proportional to the excess temperature of the surrounding. The body with initial temperature $98^\circ C$ was placed in a room of temperature $20^\circ C$. Given that body temperature drops at a rate of $24^\circ / \text{min}$. Determine the temperature of the body after 10 minutes.

16 (a) Sketch the curve $y = x(x-3)(x-6)$

(ii) Find the area bounded by the curve and the x -axis.

(b) Solve the differential equation $x^2 \frac{dy}{dx} = x^2 + y^2$, using the substitution $y = vx$.

(b) Opio walks to school at rate proportional to the square root of the distance yet to be covered. Initially he walks at a speed of 16m/s and has a distance of 0.4km to cover. Find how long it will take him to reach the school.

17 (a) Sketch on the same axes the curves $y = x^2 - 4x + 3$ and $y = 9 - x^2$.

(ii) Find the area enclosed between the two curves

(ii) Determine the volume generated when the area enclosed by the curves is rotated about the x -axis through four right angles

(b) The population of a certain city was found to increase at a rate proportional to the population present. Initially the population was 5000. After 2 years the population had tripled.

(i) Form a differential equation and solve it.

(ii) Calculate the rate at which the population was initially increasing.

(iii) How many years will it take for the population to reach 100000.

(iv) Find the number of people after 10 years.

GEOMETRY

- 18 (a) A straight line l cuts the positive axes at A and B. If the length of the AB is 10 units and the gradient of the line l is $-\frac{4}{3}$. Find the equation of the line l

(b) Determine the polar equation of the given loci.

(i) $x^2 + y^2 = 9$

(ii) $y^2 = 4x$

(c) Find the vertex, focus, directrix and the latus rectum of the parabola.

(i) $y^2 = 8x + 16$

(ii) $y^2 - 8y = 4x$

(d) Find the locus of a point that moves such that its distance

(i) from the point A(2,0) is equal to its distance from the line $x = -2$.

(ii) from the line $x = 4$ is equal to its distance from the circle $x^2 + y^2 = 1$.

- 19 (a) Determine the equation of the circle passing through the points A(-3,1), B(2,3) and C(4,-2).

(b) The circles C_1 and C_2 are given by the equations $x^2 + y^2 - 6x + 8 = 0$ and $x^2 + y^2 - 4x + 2y + b = 0$ respectively. Given that the circles are orthogonal.

(i) Determine the distance between the radii of the two circles.

(ii) Find the value of b and the radius of the circle C_2 .

(ii) (c) A tangent l is drawn from the point P(1,1) to the circle π given by the equation $x^2 + y^2 - 4x - 6y + 12 = 0$. Find the

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- (i) length of the tangent from the point $P(1,1)$ to the circle π .
- (ii) equation of the tangent l .
- 20 The point $P(ap^2, 2ap)$ lies on the parabola $y^2 = 4ax$. A line $y = mx$ parallel to the tangent meets the parabola again at Q. Given that M is the midpoint of OQ .
- (i) Find the co-ordinates of Q.
- (ii) Prove that PM is perpendicular to the y - axis.
- 21 (a) The line $2x + 2y - 3 = 0$ touches the circle π whose equation is $4x^2 + 4y^2 + 8x + 4y + c = 0$ at A .
- (i) Find radius of the circle π .
- (ii) Determine the coordinates of A.
- (b) The points $P\left(p^2, \frac{p}{3}\right)$ and $Q\left(q^2, \frac{q}{3}\right)$ lie on the parabola. M is the midpoint of PQ. If M lies on the line $y = x$, prove that the point of intersection of the tangents at P and Q lie on the curve $y^2 = \frac{1}{8}(x + y)$.
- 22 Show that the conic whose parametric equations are $x = 3 + 4\cos \theta$, $y = 1 + 3\sin \theta$ is an ellipse and find its centre and eccentricity.
- It can be proved that the line $y = mx + c$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if $c^2 = a^2m^2 + b^2$. Determine the equations of the tangent to the ellipse $\frac{x^2}{9} + \frac{y^2}{25} = 1$ drawn from the point $(-5,1)$.
- 23 (a) Determine the distance between the given point and the line
- (i) $A(3,6)$ and $3x - 4y = 12$
- (ii) $A(1,2)$ and $12x + 5y = 3$

(b) Prove that the equation of the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point

(x_1, y_1) is given by $\frac{x_1 x}{a^2} + \frac{y_1 y}{b^2} = 1$.

(c) Show that the equation of the chord joining the points whose eccentric angles are α

and β on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

$$\frac{x}{a} \cos\left(\frac{\alpha + \beta}{2}\right) + \frac{y}{b} \sin\left(\frac{\alpha + \beta}{2}\right) = \cos\left(\frac{\alpha - \beta}{2}\right).$$

Deduce the equation of the tangent at the point whose eccentric angle is ϕ .

24 (a) The normal to the rectangular hyperbola $xy = c^2$ at the $P\left(cp, \frac{c}{p}\right)$ meets the hyperbola again at the point Q . Find the

(i) Find the equation of the normal to the hyperbola at the point $P\left(cp, \frac{c}{p}\right)$.

(ii) coordinates of the point Q .

(b) The tangents at the points $P\left(cp, \frac{c}{p}\right)$ and $Q\left(cq, \frac{c}{q}\right)$ on the rectangular hyperbola intersect at R . Given that R lies on the curve $xy = 2c^2$. Show that the locus of the midpoint of PQ is given by $xy = 2c^2$.

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ALGEBRA

25 (a) Solve the equations

$$2x - y + z = 6$$

$$3x + y - z = 4$$

$$4x + y - 2z = 3$$

(b) The roots of a quadratic equation $x^2 + 2x + 8 = 0$ are α and β .

(i) Find the value of $\alpha^3 + \beta^3$

(ii) Determine the equation whose roots are $\frac{\alpha}{\beta^2 + 1}$ and $\frac{\beta}{\alpha^2 + 1}$

(c) The sum of the first n terms of the progression is $S_n = n(n + \alpha + 1)$ where α is an interger

(i) Show that the progression is Arithmetic.

(ii) if the sixth term of the progression is 14 , determine the sum of the first twenty terms and the twentieth term.

(d) A man deposits 1000/= dollars in a bank at the beginning of every year which gives a compound interest rate of 4% per annum. How much money will have accumulated by the end of the twelveth year?

26 (a) Solve the equations

$$\frac{x+y}{2} = \frac{y+z}{3} = \frac{x+z}{4} ,$$

$$x + y + z = 6$$

(b) Find the value of λ for which the equation $\frac{x-\lambda}{1-\lambda} = \frac{1}{1-\lambda} + \frac{2}{x}$ has repeated roots.

(c) The polynomial $P(x) = x^4 + ax + b$ has a factor $(x-1)^2$ find the

(i) values of a and b .

(ii) the remainder when $P(x)$ is divided by $x-1$

27 (a) Evaluate $\sqrt{8-4\sqrt{2}}$

(ii) Without using tables or calculator evaluate

$$\frac{(75)^{1/2} + (9)^{1/4}}{(27)^{1/2} - (9)^{1/4}}$$

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(b) Solve the inequalities

(i) $\frac{x^2}{x-1} \leq 3$

(ii) $\frac{9}{x^2} \geq 1$

(c) Determine the coefficient of $\frac{1}{x}$ in the expansion $\left(x - \frac{2}{x}\right)^9$.

(ii) Expand $(1 + 2x - x^2)^4$ using binomial theorem in ascending powers of x up to x^2

(iii) Expand $\sqrt{1-x}$ in ascending powers of x up to the term x^2 .

Use the expansion $(1-x)^{\frac{1}{2}}$ to find the value of $\sqrt{8}$ correct to 3 decimal places.

(d) Show that for the equations $x^2 + ax + b = 0$ and $x^2 + bx + c = 0$ to have a common root then $(b-c)^2 = (a-b)(b^2 - ac)$.

27 (a) Given that $t = x - \frac{2}{x}$, express the following in terms of t :

(i) $x^2 + \frac{4}{x^2}$

(ii) $x^3 - \frac{8}{x^3}$

Hence solve the equation $2x^4 - 11x^3 + x^2 + 22x + 8 = 0$.

(b) Prove by induction $\frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \dots + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}$

(c) Resolve into partial fractions.

(i) $\frac{8x+4}{x^2(x+2)}$

(ii) $\frac{x^2}{(1-x)(1+x^2)}$

(iii) $\frac{5x-1}{x^2-1}$

(d) The polynomial $p(x)$ leaves a remainder 3 when divided by $x+2$, a remainder 4 when divided by $x-3$. Find the remainder when the $p(x)$ is divided by $x^2 - x - 6$

28 Find n : ${}^nP_6 = 48^n C_4$

(b) Find the number of arrangements of the letters of the word REFERENDUM.

In how many ways can the letters of the word be arranged if

(i) each letter is taken one at a time.

(ii) the two letters D and M are together.

(c) A team of six delegates is to be chosen from a group of 4 men and 5 women.

In how many ways can the team be chosen if it is to comprise of

(i) an equal number of men and women.

(ii) more men than women.

29 (a) Solve the equations:

(i) $2 \log_3 x + \log_x 9 = 4$

(ii) $\sqrt{x+4} + \sqrt{x-4} = \sqrt{x-1}$

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(b) Prove by induction

$$(i) \quad 1 \times 2 + 2 \times 3 + \dots + n(n+1) = \frac{1}{3}n(n+1)(n+2).$$

$$(ii) \quad 7^n + 6n + 5 \text{ is divisible by } 6$$

(c) Expand $\sqrt{\left(\frac{1+2x}{1-2x}\right)}$ using binomial theorem in ascending powers of x up to

the term x^2 , using the substitution $x = \frac{1}{10}$ find the value of $\sqrt{6}$ correct to 3

decimal places. State the validity of the expansion.

30 (a) Given that $z = \frac{(3+2i)(2+5i)}{2-i}$, express z in

(i) cartesian form

(ii) polar form

(b) Given that $z = 3 + 2i$, find the other roots of the equation

$$z^3 - 5z^2 + 7z + 13 = 0$$

$$(i) \text{ Solve for } x \text{ and } y : \quad \frac{yi}{x+4i} = \frac{2+i}{3-2i}$$

(c) The complex number z is such that $\frac{z+2i}{z-3}$ is purely real.

Show that the locus of z is a straight line, and find its gradient.

31 (a) The complex number $z_1 = (5+2i)(6+i)$ and $z_2 = (5-2i)(6-i)$

(i) Express z_1 and z_2 in the form $a+bi$

(ii) Evaluate $z_1 z_2$ and deduce the prime factors of $28^2 + 17^2$

(b) Solve the equation $z\bar{z} + 2z = 9 + 2i$ where $\bar{z}$ is the conjugate of z

$$(ii) \text{ Simplify } \frac{\cos \frac{5\pi}{17} + i \sin \frac{5\pi}{17}}{\left(\cos \frac{4\pi}{17} - i \sin \frac{4\pi}{17}\right)^3}$$

(c) Given that $z = \cos \theta + i \sin \theta$, show that $z^n - \frac{1}{z^n} = 2i \sin n\theta$.

$$\text{Hence prove that } \sin^3 \theta = \frac{1}{4}(3 \sin \theta - \sin 3\theta)$$

- 32 (a) Given the complex numbers $z_1 = (1 + 5i)$ and $z_2 = (1 + 2i)$.

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Find the distance and angle between the two complex numbers

(b) Evaluate $(1 + i)^4 (\sqrt{3} + i)^6$

(c) Solve for x and y in the equation $\frac{x}{3+i} + \frac{yi}{1+i} = \frac{5}{1+7i}$

(d) The complex number z is such that $\left| \frac{z+3}{z-i} \right| = 2$.

Find the cartesian equation of the locus of z .

(ii) Shade the region on the complex plane satisfying the inequality $|z+3| \leq 2|z-i|$

- 33 (a) Given that $(x + yi)^2 = 5 + 12i$.

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(i) Find x and y . Hence find the two values $\sqrt{5+12i}$.

(ii) Solve the equation $z^3 - 6z^2 + 13z - 10 = 0$

(b) Use De Moivre's theorem to prove that $\tan 3\theta = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}$

(c) Sketch on the argand diagram $\arg(z - 3i) = \frac{\pi}{4}$. Determine the least value of $|z|$

VECTORS

- 34 (a) The vertices of a triangle PQR are $P(4,3,2)$, $Q(3,1,4)$ and $R(5,-2,2)$.

Prove that $\angle PQR = 90^\circ$. Find the coordinates of S if $PQRS$ is a rectangle.

(b) The lines whose vector equations are given by $r = i + 2j + pk + \lambda(2i + j - k)$ and $r = 2i - 2j + 4k + \mu(i + 2j - k)$ intersect at the point Q.

Find the

(i) values of the scalars p , λ and μ .

(ii) coordinates of Q

(iii) acute angle between the two lines.

- 35 (a) Given that $a = 2i + 3j + 2k$, $b = i + 3j + k$ and $c = i - 3j + k$.
- (i) Find the value of $|a + 2b + 3c|$
- (ii) show that the vectors a , b and c are coplanar vectors.
- (iii) Find the vector equation of the plane passing through the fixed point $P(3, 2, -2)$ and parallel to the vectors a , b and c .
- (b) The planes P_1 and P_2 whose vector equations are $3x + y + z = 7$ and $x + y - z = 3$ respectively intersect. Find the
- (i) cartesian equation of the line of intersection of the two planes.
- (ii) acute angle between the planes P_1 and P_2 .
- 36 (a) The straight line l passes through the points A and B given by the position vectors $2i + j + k$ and $3i - j + 5k$ respectively. M is the point that divides AB in the ratio 2:3. The line l meets two parallel planes Q_1 and Q_2 whose at the points M and N respectively. The vector equation of the plane Q_2 is given by $3x + 6y + 2z = 8$.
- Find the
- (i) Coordinates of M and N
- (ii) Cartesian equation of the plane Q_1
- 37 (a) Three points P , Q and R are given by the position vectors $3i + 2j - k$, $5i - j + k$ and $7i + j - 2k$ respectively. The plane π contains the three points P , Q and R . T is the foot of the perpendicular drawn from the point $A(8, 1, 3)$ to the plane π .
- (i) Show that the points P , Q and R form vertices of a triangle.
- (ii) Find the cartesian equation of the plane π .
- (iii) Determine the coordinates of T .
- 38 (a) The line l passes through the points P and Q given by the position vectors $2i + 3j + k$ and $4i - 3j + 2k$ respectively. R is a point given by the position vectors $3i + j + k$. M is the foot of the perpendicular drawn from the point R to the line l .

- Find the (i) Cartesian equation of the line l .
(ii) coordinates of M
(iii) perpendicular distance of the point R from the line l .

39 (a) The line l $\frac{x+3}{2} = \frac{y}{3} = \frac{z-2}{5}$ is parallel to the plane λ given by $2x + 4y + pz = 8$.

The plane π contains the line l and is parallel to the plane λ .

- Find the (i) value of p .
(ii) the distance between the line l and the plane λ .
(iii) Cartesian equation of the plane π .

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- 40 In a triangle OAB , $OA = a$, $OB = 3b$. P divides AB in the ratio 1:2. R is a point on OP such that $OR = \frac{1}{3}OP$. When AR is extended it meets OB at Q

(i) Show that $OR = \frac{1}{9}(2a + 3b)$

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- (ii) Express OQ in terms of a and b .

MECHANICS SEMINAR QUESTIONS

1. (a) A car covers distances of 28.6m and 35m in the fourth and eighth seconds of its motion respectively. Determine the initial speed of the car and its constant acceleration.
(b) Two points A and B are 526m apart along a straight road. A car moving along the road passes A with a constant speed of 25ms^{-1} . The car maintains this speed for 10 seconds and then decelerates uniformly to a speed of $V\text{ms}^{-1}$ for 8 seconds. The car maintains this speed until it passes point B. The total time taken by the car to move from point A to point B is 30 seconds. Sketch a velocity- time graph for the motion of the car and use it to determine the value of V .

2. (a) Car A travelling at 35ms^{-1} along a straight horizontal road, accelerates uniformly at 0.4ms^{-2} . At the same time, another car B moving at 44ms^{-1} and accelerating uniformly at 0.5ms^{-2} is 200m behind A. Find the time taken before car B overtakes car A.
(b) A body initially at rest at point O accelerates uniformly at $p\text{ms}^{-2}$. n seconds later, a second body is projected from O with a speed of $u\text{ms}^{-1}$ and accelerates uniformly at $p\text{ms}^{-2}$ towards the first particle. Given that the particles collide after a further n seconds, show that $u = 1.5pn$.

- 3(a) Derive the equation of the path of a particle projected from O at angle α to the horizontal with initial speed $u\text{ms}^{-1}$.
(b) A particle projected from point A with speed 30ms^{-1} at an angle of elevation θ , hits the ground again at the same level as A. If before landing the particle just clears the top of a tree which is at a horizontal distance of 72m from A, the top of the tree being 9m above the level AB. Calculate the possible angles of projection. (use $g = 10\text{ms}^{-2}$)

4. A body moving with acceleration $\mathbf{a} = e^{-2t}\mathbf{i} - 2\cos t\mathbf{j} + 4\sin 2t\mathbf{k}$ is initially located at a point $(2, -1, 4)$ and has a velocity of $\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$. Find the ;

- (a) Velocity of a body at any time t .
- (b) Speed of the body at time $t = \frac{\pi}{2}$ seconds.
- (c) The displacement of the body at any time

5. A particle with position vector $\begin{pmatrix} 10 \\ 3 \\ 5 \end{pmatrix}$ moves with a constant speed of 6m/s in the direction $\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$. Find its distance from the origin after 5 seconds.

(b) A particle of mass 3kg is moving on a curve described by

$\mathbf{r} = 4\sin 3t\mathbf{i} + 8\cos 3t\mathbf{j}$ where $\mathbf{r}$ is the position vector of the particle at time t .

Show that the force acting on the particle is $-27\mathbf{r}$

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6. A particle of mass 2kg moves up a line of greatest slope on a rough plane inclined at 25° to the horizontal. It is attached to a taut inextensible string which makes an angle of 30° with the plane. If the particle moves up the plane with an acceleration of 1.8ms^{-2} and the tension in the string is 20N,

- (a) Calculate the coefficient of friction between the particle and the plane.
- (b) While the particle is moving up the plane, the string is cut and the particle comes to rest. Show that the particle will remain at rest on the plane.

7. Four forces of magnitude 3N, 10N, 6N and 7N act along the sides of AB, BC, DA and DB respectively. The direction of the forces being indicated by the order of the letters, of a rectangle ABCD with sides AB=12m and BC= 5m.

(a) Taking AB and AD as x and y axes respectively, find the magnitude and direction of the resultant force.

(b) If the line of action of the resultant of the forces cuts AB produced at point M, find length MC.

8. ABCDEF is a regular hexagon of sides of length 8m, it has forces of magnitude 14N, 12N, 16N, 10N and 8N acting along the sides AB, BC, CD, ED and EF respectively. Taking AB and AE as the positive y axes. Determine the;

(a) Resultant force , its magnitude and direction.

(b) Line of action and where the resultant force crosses the x-axis.

9(a) A, B and C are three aircrafts. A has a velocity $(200\mathbf{i} + 170\mathbf{j}) \text{ ms}^{-1}$. To the pilot in A, it appears that B has a velocity of $(50\mathbf{i} + 270\mathbf{j}) \text{ ms}^{-1}$. To the pilot in B, it appears that C has a velocity $(50\mathbf{i} + 170\mathbf{j}) \text{ ms}^{-1}$. Find in vector form the velocities of B and C.

(b) Two ships A and B have the following position vectors, $\mathbf{r}$ and velocity vectors, $\mathbf{v}$ at the times stated;

$$\mathbf{r}_A = (-2\mathbf{i} + 3\mathbf{j})\text{km}$$

$$\mathbf{V}_A = (12\mathbf{i} - 4\mathbf{j})\text{Kmh}^{-1} \text{ at } 11:45\text{a.m}$$

$$\mathbf{r}_B = (-8\mathbf{i} + 7\mathbf{j})\text{km}$$

$$\mathbf{V}_B = (2\mathbf{i} - 14\mathbf{j})\text{Kmh}^{-1} \text{ at }$$

12:00noon

If the two ships do not alter their velocities, find the least distance of separation.

10.(a) To a bird flying due east at 10ms^{-1} , the wind seems to come from the south. When the bird alters its direction of flight to $\text{N}30^\circ \text{ E}$ without altering its speed, the wind appears to come from the north-west. Find the true velocity of the wind .

(b) At 8a.m two boats A and B are 5.2km apart with A due west of B, and B travelling in a direction $\text{N}20^\circ \text{W}$ at a steady speed of 13kmh^{-1} . If A travels due north at 12kmh^{-1} , determine;

(i) The path of B relative to A

(ii) The distance between A and B at 8:30am.

11(a) Show that the Centre of gravity of a uniform solid right circular cone of base radius r and height h is given by $\frac{h}{4}$ from its base.

(b) A uniform solid cone of height 12cm and base radius 3cm is formed by rotating a line about the x-axis. Find the distance of the centre of gravity of the cone from the origin.

12 .A particle performing simple harmonic motion passes through the mean point O and through points A and B in that order such that OA =10cm and AB =10cm. If the speeds at A and B are 8ms^{-1} and 6ms^{-1} respectively. Calculate the,

(i) Amplitude of motion.

(ii) Periodic time

(iii) Time taken to move from A to B directly.

13. A conical pendulum consist of a light inextensible string AB of length 50cm fixed at A and is carrying a bob of mass 2kg at B.If the bob describes a horizontal circle about a vertical through A with a constant angular speed 5rads^{-1} . Calculate;

(i) Tension in the string. (iii) Radius of the horizontal circle.

(ii) Inclination of the string to the vertical (iv) Velocity of the bob.

(v) Period of bob.

14. A non- uniform rod AB of mass 10kg has its Centre of gravity at a distance 0.25m from A. The rod is smoothly hinged at A. It's maintained in equilibrium at 60° above the horizontal by a light inextensible string tied at B and at right angle to AB. Calculate the magnitude and direction of the reaction at the hinge.

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15. A 3m long ladder rests at an angle of 60° to the horizontal against a smooth vertical wall on a rough ground. The ladder weighs 5kg and its Centre of gravity is one-third from the bottom of the ladder.

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(i) Draw a sketch diagram to show the forces acting on the ladder.

(ii) Find the reaction of the ground on the ladder.

16. A lorry of mass 4,000kg pulls a trailer of mass 1500kg ascending a slope of 1 in 10. If the resistance to motion of a lorry and the trailer is 0.2Nkg^{-1} and that their retardation is 0.8ms^{-2} . Determine the;

(a) Resistance of the lorry and trailer.

(b) Tension in the coupling between the lorry and trailer.

(c) Tractive force exerted by the lorry.

Applied Mathematics Seminar

Questions

NYAKABANGA S.S

1. (a). Two biased tetrahedrons have each of their faces numbered 1 to 4. The chance of getting any one face showing uppermost is inversely proportional to the number on it. If the two tetrahedrons are thrown and the number on the upper most face noted, determine probability that the faces show the same number.
 (b). If it is a fine day, the probability that Nathan goes to play football is $\frac{9}{10}$ and the probability that Bob goes is $\frac{3}{4}$. If it is not fine, Nathan's probability is $\frac{1}{2}$ and Bob's is $\frac{1}{4}$. Their decisions are independent. In general it is known that it is twice likely to be fine as not fine.
 (i). Determine the probability that both go to play
 (ii). If they both go to play, what is the probability that it is a fine day?

2. The table below shows the weight of seeds of a certain type of plant.

Weight (grams)	<0.10	<0.25	<0.35	<0.50	<0.60	<0.65	<0.80
frequency	2	3	5	9	3	2	3

- (a) Calculate the;

- i. Standard deviation.

- ii. Number of seedlings that weigh more than 0.57 g.

- (b) Draw a histogram and use it to estimate the modal weight.

KYAMUHUNGA S.S

3. (a). Two events A and B are such that $P(A) = \frac{8}{15}$, $P(B) = \frac{1}{3}$, and $P(A/B) = \frac{1}{5}$. Calculate the probabilities that;
 (i). Both occur
 (ii). Only one of the two events occurs
 (iii). Neither of the events occurs
 (b). A box contains 4 pink counters, 3 green counters and 3 yellow counters. Three counters are drawn at random one after the other without replacement
 (i). Find the probability that the third counter drawn is green and the first two are of the same colour.
 (ii). Find the expected number of pink counters drawn.

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4. (a). X is a discrete random variable which takes all integers from 1 to 40 such that $P(X = x) = kx$; $1, 2, 3, \dots, 40$.
 (i). Find the value of the constant k.
 (ii). Compute the standard deviation of X.
 (iii). Find $P(x < 35 / x > 20)$.
 (b). X is a random variable such that;

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$$f(x) = \begin{cases} \beta(1 - 2x); & -1 \leq x \leq 0 \\ \beta(1 + 2x); & 0 \leq x \leq 2 \\ 0 & ; \text{elsewhere.} \end{cases}$$

- (i) Sketch the pdf, $f(x)$
- (ii) Determine the value of the constant, β
- (iii) Find the mean of x
- (iv) Find the 60th percentile

5. The test marks of a certain group of candidates were distributed as follows:

Mark	$0 < x \leq 10$	$10 < x \leq 20$	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 50$	$50 < x \leq 60$	$60 < x \leq 70$	$70 < x \leq 80$	$80 < x \leq 90$
		0	0	0	0	0	0	0	0
Frequency density	0.3	0.6	0.9	1.0	1.2	1.8	1.4	1.1	0.7

- (a) Calculate the;
 - i. Mean mark.
 - ii. Standard deviation
- (b) Construct the cumulative frequency curve and use it to estimate;
 - i. Number of students who scored below 58 marks
 - ii. 25th percentile

6. (a). For a particular set of observations $\Sigma f = 20$, $\Sigma fx^2 = 16143$ and $\Sigma fx = 563$. Find the value of the;

(i). Mean

(ii). Standard deviation

(b). A set of digits consists of m zeros and n ones.

(i). Find the mean of this set of data.

(ii). Hence show that the standard deviation of the set of digits is $\frac{\sqrt{mn}}{m+n}$

(c). A physics student measured the time required in seconds for a trolley to run down slopes varying gradients and obtained the following results.

32.5, 34.5, 33.5, 29.3, 30.9, 31.8. Calculate the mean time and standard deviation.

7. (a). Wear test on 100 components gave the following grouped frequency distribution of the life length.

Life length (x hours)	Number of components
$500 \leq x < 530$	15
$530 \leq x < 550$	24
$550 \leq x < 570$	33
$570 \leq x < 600$	21
$600 \leq x < 650$	7

Use linear interpolation to estimate the probability that a component drawn at random from the 100 has a life length between 540 and 580 hours.

(b). The duration of 60 telephone calls are summarized in the table below.

Duration (minutes)	0 -	9 -	18 -	27 -	36 -	45 -
Number of calls	6	10	21	20	3	0

Use linear interpolation to estimate the probability that the duration of a call, selected at random from the 60 calls, exceeds 30 minutes.

8. (a). Metal rods produced by a machine have lengths that are normally distributed. 20% of the rods are rejected for being shorter than the minimum acceptable length of 35mm. Given that 65% of the metal rods produced are between 35mm and 45mm;

- Calculate the mean and variance of the lengths of the metal rods produced
- If 10 rods are chosen at random from a batch produced by the machine, find the probability that at least one of them has a length greater than 40mm.

(b). A certain tribe is distinguished by the fact that 45% of the males have six toes on their right foot. Find the probability that in a group of 200 males from the tribe, more than 97 have six toes on their right foot.

(c). The heights of students in a particular school are distributed with mean μ and standard deviation σ . On the basis of the results obtained from a random sample of 100 students from the school the 95% confidence interval for the μ was calculated and found to be [177.22cm, 179.18cm]. Calculate the value of the sample mean $\bar{x}$ and value of σ

9. Eight candidates seeking admission to a University course sat for a written and oral test. The scores were as shown in the table below.

Written X	55	54	35	62	87	53	71	50
Oral Y	57	60	47	65	83	56	74	63

- Draw a scatter diagram for this data
- Draw a line of best fit on your scatter diagram
- Use the line of best fit to estimate the value of Y when X = 70.
- Calculate the rank correlation coefficient. Comment on your result.

10. The table below shows the expenditure of a restaurant for the year 2014 and 2016.

Item	Price (shs)		Weight
	2014	2016	
Milk (per litre)	1000	1300	0.5
Eggs (per tray)	6500	8300	1
Sugar (per kg)	3000	3800	2
Blue band	7000	9000	1

Taking 2014 as the base year, calculate for 2016 the;

- Price relative for each item

- (ii) Simple aggregate price index
- (iii) Weighted aggregate price index and comment on your result.
- (iv) In 2016, the restaurant spent shs. 4,500 on buying these items. Using the index obtained in (iii), find how much the restaurant could have spent in 2014.

11. (a). The table below shows the distribution of weights of a group of animals.

Mass (kg)	Frequency
21 – 25	10
26 – 30	20
31 – 35	15
36 – 40	10
41 – 50	30
51 – 65	45
66 – 75	5

- (i) Construct a histogram for the above data and use it to estimate the mode.
 - (ii) Calculate the median for the above data.
- (b). The distribution below shows the weights of babies in Gombe hospital
3, 5, 3, 9, 6, 8, 20, 19, 24, 14, 12. Find the;
- i. Upper quartile
 - ii. Lower quartile
 - iii. Median
 - iv. Variance
 - v. Standard deviation

12. (a). A continuous random variable X has a probability density function $f(x)$ given by;

$$f(x) = \begin{cases} \frac{k}{x(4-x)} & 1 \leq x \leq 3 \\ f(x) = 0 & \text{otherwise} \end{cases}$$

- (i) Show that $k = \frac{2}{\ln 3}$
 - (ii) Calculate the mean and variance of X
- (b). The probability density function of a random variable X is given by;

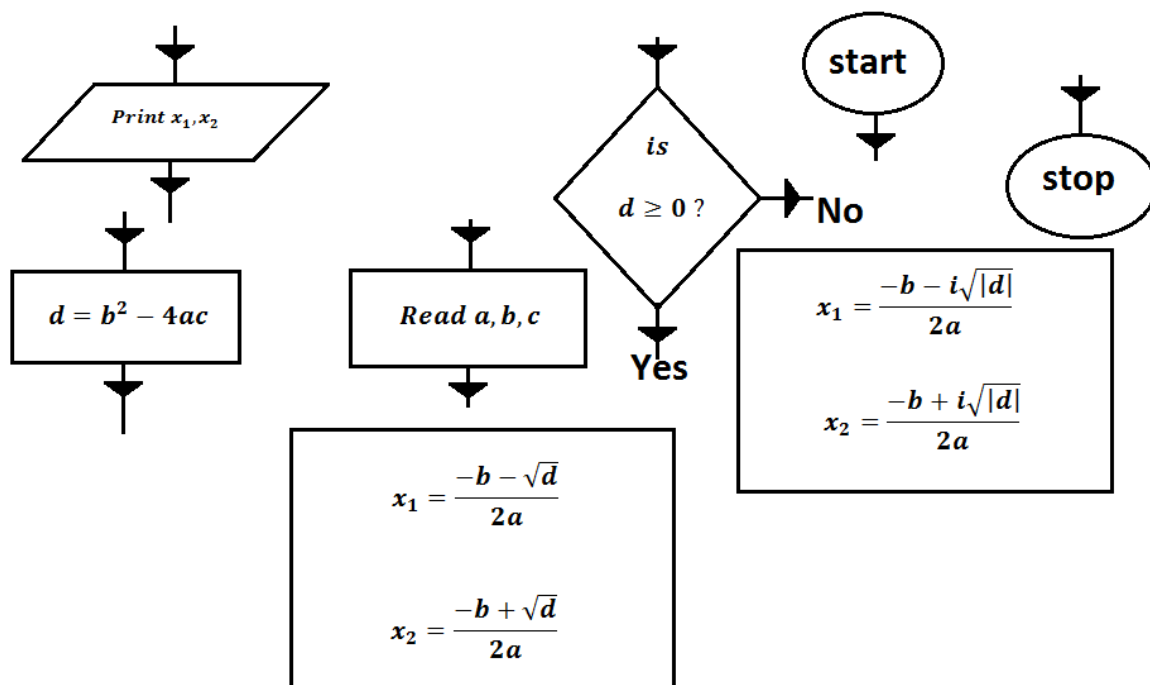
$$f(x) = \begin{cases} c(x+2), & -1 \leq x \leq 0 \\ 2c(1-x), & 0 \leq x \leq 2 \\ 0, & \text{elsewhere} \end{cases}$$

- (i) Sketch $f(x)$ and find the value of c
- (ii) Find the probability $P(|x-1| < 0.5)$

13. The probabilities of events A and B are $P(A)$ and $P(B)$ respectively. $P(A) = \frac{5}{12}$, $P(A \cap B) = \frac{1}{6}$ and $P(A \cup B) = q$. Find in terms of q ,
- $P(B)$
 - $P(A/B)$
 - Given that A and B are independent, find the value of q
- (b). A box contains 8 green and 4 red apples. Five apples are selected at random from the box in succession without replacement. What is the probability that three of them are green?
14. (a). Use the trapezium rule to estimate the area of 5^{2x} between the x-axis, $x=0$ and $x=1$, using 5 sub-intervals. Give your answer correct to 3dps.
- (b). Find the exact value of $\int_0^1 5^{2x} dx$
- (c). Determine the percentage error in the two calculations in (a) and (b) above.
- (d). How should the error be reduced.
15. (a). Show graphically that the equation $x^3 + 5x^2 - 3x - 4 = 0$ has roots between 0 and -1.
- (b). Use the Newton Raphson method to calculate the root of the equation in (a) correct to 2dps.
16. If the numbers x and y are approximations with errors Δx and Δy respectively;
- Show that the maximum absolute error in the approximations of x^2y is given by $|2xy\Delta x| + |x^2y|$. Hence find the limits within which the true values of x^2y lies given that $x = 2.8 \pm 0.016$ and $y = 1.44 \pm 0.008$.
 - Given two numbers $x = 3.815$ and $y = 2.43$, find the absolute error in the quotient $\frac{y}{x}$, truncate to 3dps. Hence, find the least and greatest value of $\frac{y}{x}$ correct to two decimal places.
17. (a). Given that $y = \sec(45^\circ \pm 10\%)$, find the limit within which the exact value of y lies
- (b). Construct a flow chart that computes and prints the average of the squares of the first six counting numbers. Perform a dry run for your flowchart.
18. The method for solving the quadratic equation $ax^2 + bx + c = 0$ is described in the following parts of the flow chart.

KIRUGU S.S

RUYONZA S.S



- (i) By re-arranging the given parts, draw a flow chart that shows the algorithm for the described method.
- (ii) Perform a dry run for $x^2 - 4x + 13 = 0$, and state the roots of the equation.

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END